THE STRUCTURE CALCULATION OF K^-pp WITH Λ^*p MODEL

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Abstract

The purpose of this research is to investigate the structure of K^-pp system by using Λ^*p model, where Λ^* is a resonance state of K^-p system. To determine the binding energy of this system, Schrödinger equation is solved in two body system, which consists of Λ^* and proton. In our calculation, the Gaussian basis wave function is used to solve the Schrödinger equation. Phenomenologically constructed potential of Λ^*p interaction is used in this calculation. The experimental value for the binding energy of Λ^* is 27 MeV. By using theYamazaki and Akaishi phenomenological potential, the calculated result of the binding energy of K^-pp system from the Λ^*p threshold is 21.17 MeV and level width is 61 MeV. The binding energy of K^-pp system from $(K^- + p + p)$ threshold is $(\mathbf{21.17 + E_{\Lambda^*}}) = \mathbf{48.17\ MeV.}$ By using the DISTO experimental data, the calculated binding energy for K^-pp system from $\Lambda^* p$ threshold is 78.0457 MeV and level width is 118.4 MeV. The binding energy of K^-pp system from $(K^- + p + p)$ threshold is $(78.04 + E_{\Lambda^*}) = 105.04$ MeV. Thus, the calculated results for binding energy of K^-pp system are in good agreement with the calculated result of YA and experimental result of DISTO.

Keywords Power inverse iteration method, Two body system, Three body system.

Introduction

A "K^-pp"-like structure has observed in the d (π^+, K^+) reaction at 1.69 GeV/c. In this reaction $\Lambda(1405)$ hyperon resonance is expected to be produced as a doorway to form the $K^- pp$ through the $\Lambda^* p \rightarrow K^- pp$ process.

Since this is a three-body system, several groups calculated the binding energy and width of K^-pp by applying various few-body calculation techniques such as variational and Feddeev type calculations. The obtained binding energies are scattered in a broad range: 10-20 MeV for shallow potential cases and 50-100 MeV for deep cases. The width would be as wide as 70 MeV because of the strong $\overline{K}N - \pi\Sigma$ coupling. In addition there could be non-mesonic absorption contributions of $\overline{K}NN \rightarrow \Lambda(\Sigma)N$.

The first experimental evidence of the K^-pp bound state was reported by the FINUDA collaboration in the stopped K^- absorption reactions on ^{6}Li, ^{7}Li and ^{12}C targets. They observed a lot of Λp pairs emitted in back-to-back, and found the invariant mass of the pair significantly lower than K^-pp mass threshold.

The binding energy of $115^{+6}_{-5}(\text{stat.})^{+3}_{-4}(\text{syst.})\,\text{MeV}$ and the decay width of $\Gamma = 67^{+14}_{-11}(\text{stat.})^{+2}_{-3}(\text{syst.})\,\text{MeV}$ were obtained. However, there was a theoretical criticism to interpret the observed structure as the K^-pp bound state. Experimental evidence was reported by

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the DISTO collaboration. They measured the missing-mass and invariant mass spectra in an exclusive reaction of $pp \rightarrow K^{+}\Lambda p$ at 2.85 GeV. The binding energy of 103 ± 3 (stat.) ± 5(syst.) MeV and the width of 118 ± 8 (stat.) ± 10 (syst.) MeV were obtained. However, they did not observe the signal at 2.50 GeV, may be due to the less production cross section of Λ (1405) at this energy.

The Λ(1405) is conventionally assumed to be a well-established resonance and to represent the I = 0, strangeness = –1, $J^{p} = \frac{1}{2}^{-}$ state within the L = 1 supermultiplet of the three-quark system. Lying roughly 30 MeV below the $\overline{K}N$ threshold, the resonance can only be observed directly in the $(\sum\pi)^{0}$ system of final states of production experiments. It was first reported by Alston et al. in the reaction $K^{-}p \rightarrow \sum 3\pi$ at 1.15 GeV/c and subsequently "seen" in several other experiments. However these observations suffered from

a. low statistics,

b. difficulties with the reconstruction of final states involving ∑ decays, and

c. the uncertainly in the removal of backgrounds, particularly that from

$\Sigma(1385) \rightarrow \sum\pi$ whose rate was badly know.

In fact, none of the experiments during the period 1962-1972 were able to demonstrate a convincing signal, to measure the precise mass and width, or to determine the quantum numbers.

The only experiment which has reported a respectable signal for Λ (1405) is that of Thomas et al. in the reaction $\pi^{-}p \rightarrow \sum\pi K$ at 1.69 GeV/c. Subsequently, Chao et al. demonstrated the importance of such data in constraining the multichannel analyses. Very recently, Dalitzetal have restated the debate on the nature of the Λ (1405) resonance and asked for more precise data on the production line-shape of the $\Lambda(1405) \rightarrow \sum\pi$. Currently, it is not known whether the interpretation of the Λ (1405) as a three-quark state is or is not consistent with its appearance as an unstable $\overline{K}N$ bound state.

Two- Body Calculations

In order to calculate the structure of two-body quantum system we solved Schrödinger equation.

$$HU = EU$$

$$(H_0 + V)\, U = EU$$

Where,

H= Hamiltonian operator

E= Energy eigen value

U = Eigen vector

H_0 = Kinetic energy operator

V = Potential energy operator

Since, the interaction between Λ^* and proton is central force, so we choose the spherical coordinate for the wave function

$$\Psi(\vec{r}) = R(\vec{r})\Theta(\theta)\Phi(\varphi)$$

where,

$R(\vec{r})$ is a radial part and $\Theta(\theta)\Phi(\varphi)$ is an angular part .

$Y_l^m(\theta,\varphi) = \Theta(\theta)\Phi(\varphi)$ is spherical harmonic which are well known.

We have to solve only the radial part $R(r)$ with the local potential type.

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}U(r) = E\,U(r) \quad \text{----------(1)}$$

Where, $U(r) = r\,R(r)$

The reduced mass, $\mu = \dfrac{M_{\Lambda^*} M_p}{M_{\Lambda^*} + M_p}$

M_{Λ^*} =mass of Λ^*

M_p =mass of proton

Gaussian Basic Treatment

To solve the above equation, we will use the Gaussian Basis wave function.

$$U(r) = r^{\ell+1}\sum_j c_j\, e^{-\left(\frac{r}{b_j}\right)^2} \quad \text{----------(2)}$$

Where, c_j's are expansion coefficients and b_j's are range parameters which are adjusted in the calculations with

$$b_{j+1} = c\,b_j$$

$$b_N = c^{N-1}\,b_1$$

$$c = \left(\frac{b_N}{b_1}\right)^{\frac{1}{N-1}}$$

The Schrödinger equation becomes

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}\sum_j c_j\, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} = E\sum_j c_j\, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2}$$

$$\sum_j H_{ij}\,c_j = E\sum_j N_{ij} c_j$$

$$H_{ij} = T_{ij} + A_{ij}^{\ell} + V_{ij}$$

H_{ij} is the Hamiltonian matrix element

$$[H][c] = E[N][c]$$

Where, [H] and [N] are square matrices. [c] is column matrix. N_{ij}, T_{ij} and A_{ij}^{ℓ} are analytically solved by using standard integral form.

Calculation of Eigen Value

In the previous section, the Schrödinger equation can be written as the following matrix equation.

$$[H][c] = E[N][c]$$

$$[N]^{-1}[H][c] = E[c]$$

$$[A][c] = E[c]$$

Which is an eigen value equation with

$[A] = [N]^{-1}[H]$, $[N]$ is called the norm matrix and [H] is the Hamiltonian matrix. We are going to use the power inverse iteration method to calculate eigen values E with corresponding eigen vectors c.

Calculation of the Physical Quantities

We normalized wave function as $\int u^{*}\, u\, dr = 1$

$u = A\sum_{j} c_j\, r^{\ell+1}\, e^{-\left(\frac{r}{b_j}\right)^2}$ Where, A is the normalization constant.

$$\int A^{*} \sum_{i} c_i^{*}\, r^{\ell+1}\, e^{-\left(\frac{r}{b_i}\right)^2}\; A\sum_{j} c_j\, r^{\ell+1}\, e^{-\left(\frac{r}{b_j}\right)^2} dr = 1$$

$$|A|^2 = \sum_{i}\sum_{j} c_i^{*} c_j\, N_{ij} = 1$$

$$|A| = \frac{1}{\sqrt{\sum_{i}\sum_{j} c_i^{*} c_j\, N_{ij}}}$$

$$\mathbf{u}(\vec{\mathbf{r}}) = \mathbf{A}\sum_{\mathbf{j=1}}^{\mathbf{N}} \mathbf{r}^{\ell+1} \mathbf{c_j}\, \mathbf{e}^{-\left(\frac{\mathbf{r}}{\mathbf{b_j}}\right)^2} \quad \text{----------(3)}$$

With this normalized wave function, we have calculated the physical quantities which are average kinetic energy $\left\langle \hat{T} \right\rangle$, average centrifugal potential energy $\left\langle A_{ij}^{\ell} \right\rangle$ and average potential energy $\left\langle \hat{V} \right\rangle$. To find the average kinetic energy, we have the relation such that

$$\left\langle \hat{\mathbf{T}} \right\rangle = \left\langle \mathbf{u} \left| \hat{\mathbf{T}} \right| \mathbf{u} \right\rangle \text{----------} (4)$$

$$= \int u^{*} \hat{T} u \, dr$$

$$\left\langle \hat{T} \right\rangle = \int A^{*} \sum_{i} c_{i}^{*} r^{\ell+1} e^{-\left(\frac{r}{b_i}\right)^2} \left\{ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} \right\} A \sum_{j} c_j r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} \, dr$$

$$= |A|^2 \sum_{i} \sum_{j} c_i^{*} \, c_j T_{ij} \qquad \text{Where, } T_{ij} = -\frac{\hbar^2}{2\mu} \int r^{\ell+1} e^{-\left(\frac{r}{b_i}\right)^2} \frac{d^2}{dr^2} r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} \, dr$$

The average centrifugal potential energy, the relation can be expressed as

$$\left\langle \mathbf{A}_{\mathbf{ij}}^{\ell} \right\rangle = \left\langle \mathbf{u} \left| \mathbf{A}_{\mathbf{ij}}^{\ell} \right| \mathbf{u} \right\rangle \text{----------} (5)$$

$$= \int u^{*} \, A_{ij}^{\ell} \, u \, dr$$

$$= \int A^{*} \sum_{i} c_i^{*} r^{\ell+1} e^{-\left(\frac{r}{b_i}\right)^2} \left\{ \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r^2} \right\} A \sum_{j} c_j \, r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} \, dr$$

$$= |A|^2 \sum_{i} \sum_{j} c_i^{*} c_j A_{ij}^{\ell} \qquad \text{Where,} \quad A_{ij}^{\ell} = \int r^{2(\ell+1)} \left\{ \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r^2} \right\} e^{-\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right) r^2} \, dr$$

The average potential energy numerically, the relation can be expressed as

$$\left\langle \hat{\mathbf{V}} \right\rangle = \left\langle \mathbf{u} \left| \hat{\mathbf{V}} \right| \mathbf{u} \right\rangle \text{----------} (6)$$

$$= \int u^{*} \hat{V}(r) u \, dr$$

$$= \int |A|^2 \sum_{i} c_i^{*} \, r^{\ell+1} e^{-\left(\frac{r}{b_i}\right)^2} \left(\bar{v}_0 + i \bar{\omega}_0 \right) \left(\frac{r}{b} \right)^2 e^{-\left(\frac{r}{b}\right)} \sum_{j} c_j r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr$$

$$= |A|^2 \sum_{i} \sum_{j} c_i^{*} \, c_j \int r^{2(\ell+1)} \, V(r) e^{-\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right) r^2} dr$$

The relation of the root mean square radius,

$$\left\langle \bar{\mathbf{r}}^{\mathbf{2}} \right\rangle = \int \mathbf{u}^{*} \, \mathbf{r}^{\mathbf{2}} \, \mathbf{u} \, \mathbf{dr} \text{----------} (7)$$

$$\left\langle \bar{r}^2 \right\rangle = \int A^* \sum_i c_i^* \, r^{\ell+1} e^{-\left(\frac{r}{b_j^2}\right)} r^2 A \sum_j c_j r^{\ell+1} e^{-\left(\frac{r}{b_j^2}\right)} \, dr$$

$$\sqrt{\left\langle \bar{r}^2 \right\rangle} = \sqrt{|A|^2 \sum_i \sum_j c_i^* c_j \frac{(2\ell+3)!!}{2^{\ell+3}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{\ell+\frac{5}{2}}}}$$

Interactions

The elementary $\bar{K}N$ and NN interactions deduced semiempirically to obtain not only the binding energy and width but also the spatial and momentum distribution of the individual particles. Three-body calculations by showing that the $\bar{K}N$ complex potential, which is transformed from coupled-channels interactions, has very little energy dependence. Furthermore the result remains unchanged, even when the $\bar{K}N$ and NN interactions to vary in a wide range, as long as they reproduce the energy and width of $\Lambda(1405)$. The predicted K^-pp is a compact nuclear system with a binding energy around 50MeV and a root-mean-square (rms) p-p distance of 1.9 fm. The K^-p pair (quasi- Λ^*) behaves like an atomic unit in a "molecule" of K^-pp, similarly to the mechanism of the Heitler-London scheme. Namely, a super strong nuclear force is caused by a migrating real $\bar{K}$ meson.

The Λ (1405) resonance state the I = 0 is bound state of $\bar{K}N$. Through the main part of is article the "classical" experimental values for the binding energy and width,

$$-B_k = E_{\bar{K}N}^{I=0} = -27 \text{ MeV}, \Gamma = 40 \text{ MeV}$$

The Λ^* data combined with the kaonic hydrogen shift (yielding a_{K^-p}) and Martin's $\bar{K}N$ scattering length ($a^{I=0}$ and $a^{I=1}$),

$$a_{K^-p} = (-0.78 \pm 0.15) + i\,(0.49 \pm 0.28) \text{fm}$$

$$a^{I=0} = (-1.70 \pm 0.07) + i\,(0.68 \pm 0.04) \text{fm}$$

$$a^{I=1} = (0.37 \pm 0.09) + i\,(0.60 \pm 0.07) \text{ fm}$$

Where used in a coupled-channels calculation to deduce the $\bar{K}N$ interactions of the following forms

$$v_{\bar{K}N}^I = v_D \exp\left[-\left(\frac{r}{b}\right)^2\right],$$

$$v_{\bar{K}N,\pi\Sigma}^I = v_{C_1} \exp\left[-\left(\frac{r}{b}\right)^2\right],$$

$v^{I}_{\bar{K}N,\pi\Lambda} = v_{C_2} \exp\left[-\left(\frac{r}{b}\right)^2\right]$, Where b = 0.66 fm and

$v_D^{I=0} = -436\,\text{MeV}, v_{C_1}^{I=0} = -412\ \text{MeV}, v_{C_2}^{I=0} = \text{none}, v_D^{I=1} = -62\,\text{MeV},$

$v_{C_1}^{I=1} = -285\,\text{MeV}$, $v_{C_2}^{I=1} = -285\,\text{MeV}$.The two interactions, $v^{I}_{\pi\Sigma}(r)$ and $v^{I}_{\pi\Lambda}(r)$ are taken to be vanishing to simply reduce the number of parameters. This is justified because they are almost irrelevant in describing the $\bar{K}$ bound states.

The above coupled-channels interactions were used to derive equivalent single-channel $\bar{K}N$ potentials with imaginary parts in energy-independent forms, which is an appropriate way to obtain the decaying state of Kapur-Peierls as discussed below. The obtained complex potentials are

$$v^{I=0}_{\bar{K}N}(r) = (-595 - i83)\ \exp\left[-\left(\frac{r}{0.66}\right)^2\right],$$

$$v^{I=1}_{\bar{K}N}(r) = (-175 - i105)\ \exp\left[-\left(\frac{r}{0.66}\right)^2\right],$$

in units of MeV and fm. The same rage is assumed for I=0 and I=1. The interaction strength (V_0) and the rage (b) can be determined simultaneously because B and a_{K^-p} have different dependences on V_0 and b. Semiempirical $\bar{K}N$ interaction is consistent with the theoretically derived ones from meson-exchange and from chiral dynamics.

Nothing that the parameter b in the above Gaussian distribution is related to the rms distance R as $b = \sqrt{\frac{2}{3}}R = 0.816R$, the observed proton rms radius (R_p= 0.862 fm) to give a range parameter b = 0.70 fm, which is compatible with rage parameter (0.66 fm). The $\bar{K}N$ scattering amplitude by changing the range parameter b. The real and imaginary parts with b = 0.7 fm reproduce the chiral dynamics result very well, in spite of the strong claim by Oset and Toki that AY's scattering amplitudes are too large compared with those obtained from the chiral unitary approach of Oset and Ramos. Thus, the interaction rage deduced and used in AY is fully justified.

Table 1 Calculated potential parameters (V_0 and W_0 in MeV), energies (E_{K^-pp})and widths (Γ_{K^-pp}) of K^-pp in MeV, and the I=0 scattering length in fm with varied $\bar{K}N$ rage (b in fm), while reproducing Λ (1405).

b	V_0	W_0	E_{K^-pp}	Γ_{K^-pp}	$A^{I=0}$ (fm)
1.0	-316.5	-62.0	-49.5	66.5	-1.95+i0.45
0.9	-368.7	-67.0	-49.0	65.7	-1.89+i0.44
0.8	-439.6	-73.0	-48.3	64.4	-1.82+i0.44
0.7	-540.0	-81.0	-47.3	62.9	-1.75+i0.43
0.6	-689.5	-91.0	-45.8	60.3	-1.69+i0.43
0.5	-929.7	-105.0	-44.0	57.4	-1.62+i0.42
0.4	-1358.0	-128.0	-42.1	54.9	-1.55+i0.42
0.3	-2250.0	-162.0	-40.1	51.3	-1.48+i0.42

Results and Discussions

The structure of K^-pp nuclear cluster comprehensively by solving three-body system exactly in a variational method starting from the $\Lambda(1405)$ resonance $(\equiv \Lambda^*)$ is a K^-p bound state. The prediction for the presence of K^-pp as a compact bound system with M = 2322 MeV/c^2, B_k = 48 MeV and Γ = 60 MeV remains unchanged by varying the $\bar{K}N$ and NN interactions widely as far as Λ (1405). The Λ^*- p system exists a compact doorway state propagating to $K^- pp$ $\left(\mathbf{R_{\Lambda^* p} \approx 1.67\,fm}\right)$.

The potential used in the calculation is $V^{opt}(r) = (\bar{v}_0 + i\bar{\omega}_0)\left(\frac{r}{b}\right)^2 \exp -\left(\frac{r}{b}\right)$

In Y-A potential, the value of $\bar{\mathbf{v}}_0 = -\mathbf{227.0\,MeV}$

$$\bar{\omega}_0 = -\mathbf{106.0\,MeV}$$

b =0.30fm

By using Y-A potential, the calculated value of binding energy and level width for Λ^* p system are represented in table (2).

In DISTO potential, the value of $\bar{\mathbf{v}}_0 = -\mathbf{400.0\,MeV}$

$$\bar{\omega}_0 = -\mathbf{162.0MeV}$$

b = 0.3 fm

By using DISTO (Defense Industrial Security Education & Training Office) experimental data, the calculated value of binding energy and level width forΛ^* p system are represented in table (3).

Then, average kinetic energy, average potential energy and root mean square radius values are calculated.

By using Y-A phenomenological potential and the DISTO experimental data, the calculated results of average kinetic energy, theaverage potential energy and the root mean square radius for Λ^* and p system are represented in table (4).

In the three body K^-pp system, Yamazaki-Akaishi calculated rms value between N and the $\overline{K}N$ is 1.67 fm. The calculated rms values are different because of using the different potential models.

Table 2 The calculated results of binding energy and level width of Λ^*p system for Yamazaki Akaishi potential.

Λ^*p potential	BE (MeV)	Level Width (MeV)
Y-A phenomenological potential [T.Yamazaki]	-21.00	61.00
Our Calculated Result	-21.168	60.76

Table 3 The calculated results of binding energy and level width of Λ^*p system for DISTO experimental data.

Λ^*p potential	BE (MeV)	Level Width (MeV)
DISTO Experimental Data [M. Maggiora et al]	-78.00	108.00
Our Calculated Result	-78.05	108.52

Table 4 Calculated results of kinetic energy, potential energy and root mean square radius For Λ^*p system.

	Kinetic Energy (MeV)	Potential Energy (MeV)	RMS (fm)
Y-A phenomenological Potential [T.Yamazaki]	43.90	-65.06	1.46
DISTO Experimental Data [M. Maggiora et al]	68.27	-146.32	1.11

Conclusion

Thus, the calculated results for binding energy of K^-pp system are in good agreement with the calculated result of YA and experimental result of DISTO. The calculated rms values are different because of using the different potential models. The Λ (1405) plays an essential role in forming the $\overline{K}$ nuclear clusters (KNC). A new window to Nuclear Physics would be opened up through the investigation of KNC.

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